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INVESTIGATION OF CONTROL LAW FOR ACTIVE FLUTTER SUPPRESSION

Wang Weiping, Gu Zhongquan and Zhu Demao

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16. Abstract

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INVESTIGATION OF CONTROL LAW FOR ACTIVE FLUTTER SUPPRESSION

Wang Weiping, Gu Zhongquan and Zhu Demao
Nanjing Aeronautical College

/566

Abstract

This paper studies the technology of active flutter suppression by using the wind-tunnel model of a delfa wing with low-aspect ratio as research specimen. The researches include the establishment of mathematical model for a servo-aeroelastic system and the synthesis and analysis of sub-optimal output feedback control laws for active flutter suppression.

Explanation on Symbols

A -- Dynamic matrix of controller;

b_o -- Semi-root chord length of the wing;

B, B_o, B_c -- Input matrices of ccntroller;

C -- Output matrix of controller;

C_p, M_p, K_p -- Structure damping, mass and rigidity matrices;

F_a -- Dynamic matrix of control system;

F_s -- Dynamic matrix of control system;

G_a -- Input matrix of control system;

G_v -- Input matrix of servoaerodynamic elastic system;

G_w -- Input matrix of gas flow disturbance;

J_s -- Secondary type performance Index;

R_w -- Intensity of air flow disbance white noise;

r -- Number of acceleration sensors;

s, $\bar{s} + SV/b_o$ -- Dimension and dimensionless Laplace variables;

u, u_o -- Input signal and input noise;

u_c -- Output signal of controller;

V -- Air flow velocity;

w -- Air flow disturbance white noise;

w_z -- Air flow disturbance velocity;

x_a -- Status vector of closed-loop control system;

x_c -- Status vector of controller;

x_z -- Status vector of air flow disturbance model;

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* Numbers in the margin indicate pagination in the foreign text.

N_c -- Number of orders of controller;
N_f -- Number of structure oscillation types;
N_w -- Number of order of aerodynamic disturbance model;
m -- Number of control planes;
Q_1, Q_2 -- Output and input weighted matrices;
$q = (1/2)\rho V^2$ -- Dynamic pressure of air flow;
R_{u_o}, R_{y_o} -- Intensities of input and measured noise.
x_p -- Vector of coordinate in the broad sense;
x_s -- Status vector of servo-aerodynamic elastic system;
y_o -- Measur d noise vector;
y_s -- Output signal vector;
β -- Deflection angle of control plane;
ρ -- Density of air flow;

I. Foreword

As indicated by research for more than a decade, there are many advantages to apply the active control technique and aerodynamic control plane to control aeronautical flutter compared to the classical suppression technique (such as additional distributed weight) for passive flutter. At present, this has become a most promising new technique in aircraft design with layout under control. Derivation of the control law is one key sector of the active flutter suppression /567
technique; however, the linear secondary Gauss theory is currently a relatively popular design method [1] of the control law. This method, however, suffers from two basic problems: (1) selection and iterative substitutional convergence of the initial value of the controller; and (2) theoretically, it cannot ensure closed-loop stability of the control system at design velocities (not to mention the stability throughout the flight envelope). In solving this problem, the paper proposes a slowly-varying parametric suboptimal output feedback control law with inequality constraint. Using a low speed wind tunnel model with a delta wing having a small aspect ratio, the authors used ground experiments to find the low-order mode parameters and the transfer function of the servorudder mechanism. By using a subsonic dipole network, the nonsteady coefficient of the aerodynamic force in the broad sense is calculated. In addition, the Roger approximation is used for

rational fractinalization, thus establishing the status equations of the servoaerodynamic elastic system. Next, the authors combine the frequency band method and modern control theory; that is, first of all based on the frequency band method, the parameters of the filter in the controller are determined. Then, applying the slowly varying design method suboptimal output feedback control law, other control law parameters are determined. In addition, the paper gives the result of analyzing the primary characteristics of suppressing the control system by active flutter of the wing model.

II. Wind Tunnel Experimental Wing Model

The research object of this paper is a low-speed wind tunnel model of a delta wing with a small aspect ratio; refer to Fig. 1 for schematic diagram of the model configuration and structure. With a cantilever support, the model was installed in the wind tunnel. By applying a mode status testing technique, the authors determined the structural mode status parameters of the first five orders. Table 1 lists the corresponding mode status frequency and damping.

TABLE 1. STRUCTURAL MODE PARAMETERS OF A FLIGHT-VEHICLE WING

	a 第 1 阶	b 第 2 阶	c 第 3 阶	d 第 4 阶	e 第 5 阶
f 模态频率(Hz)	4.36	7.26	12.9	20.24	21.0
g 模态阻尼比	0.0091	0.0081	0.0080	0.0168	0.0121

Key: a. First order; b. Second order; c. Third order; d. Fourth order; e. Fifth order; f. Mode status damping (Hz); g. Ratio of mode status damping.

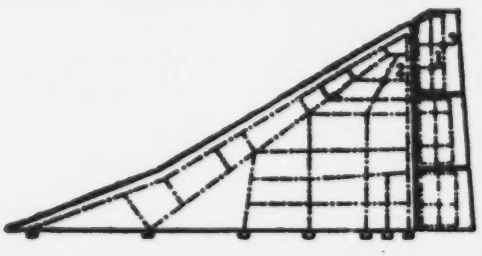

Fig. 1. Wind Tunnel Test Wing Model.

The intermediate aileron of the model is used as an aerodynamic control plane, driven by a hydraulic rudder mechanism. The authors applied a fast sine scanning test technique to deter-

mine the frequency response characteristics of the rudder mechanism, and utilized the transfer function simulation method to obtain the transfer function of the rudder mechanism [2].

$$\frac{\beta(s)}{u(s)}=\frac{4.769\times10^{5}}{s^{4}+241s^{3}+1.919\times10^{5}s+3.212\times10^{7}}\ \text{rad/V} \qquad (1)$$

Using the subsonic dipole network method, the nonsteady aerodynamic force in the broad sense of a wing model (of flight vehicle) was calculated. In addition, by applying Roger's approximation, rational fractionalization was conducted, thereby obtaining the status three-dimensional equations and output equations of the servo-aerodynamic elastic system (refer to Annex A).

$$\dot{x}_1=F_1x_1+G_1u+G_1w \qquad (2)$$

$$y_1=H_1x_1+y_0 \qquad (3)$$

As indicated by the calculated result (referring to Fig. 2) for the open-loop flutter property of the wing model, the flutter status is the coupling flutter of the first-order bending of the wing and elastic deflection of the aileron along the outer side of the wing; the critical velocity is 19.8 meters per second, and the flutter frequency is 6.24 Hertz. However, the wind tunnel flutter experimental result for the model is as follows: flutter critical velocity, 20.6 meters per second and flutter frequency, 6.2 Hertz. The calculation error is less than 4 percent.

III. Design Method of Suboptimal Output and Feedback Control Law /568

Assume that the dynamic equation (referring to Fig. 3) of the controller is:

$$\left.\begin{aligned}\dot{x}_c&=Ax_c+By_1\\ u_c&=Cx_c\end{aligned}\right\} \qquad (4)$$

The augmented status equation of the closed-loop control system can be obtained:

$$\dot{x}_0=F_0x_0+G_0\eta_0 \qquad (5)$$

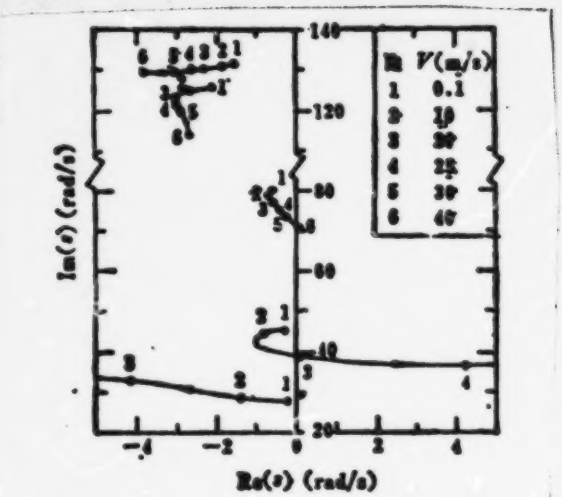

Fig. 2. Open-loop Root-locus Diagram of Wing in a Flight Vehicle.

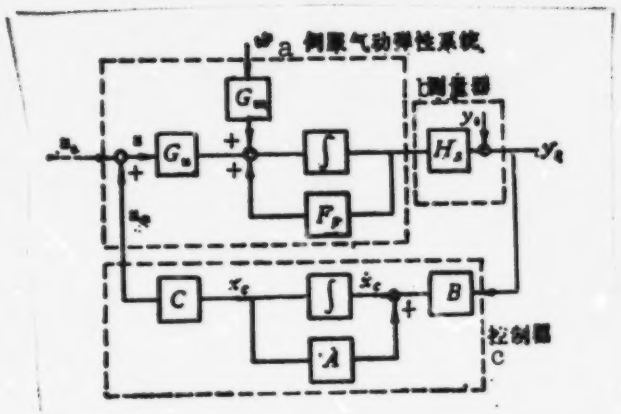

Fig. 3. Block Diagram of Closed-loop Control System.
Key: a. Servoaerodynamic elastic system; b. Instrumentation; c. Controller.

In the equation,

$$x_s=\left\{\frac{x_f}{x_c}\right\},\quad F_s=\left[\begin{array}{c|c}F_f & G_cC\\ \hline BH_f & A\end{array}\right]$$

$$G_s=\left[\begin{array}{c|c|c}G_a & G_w & 0\\ \hline 0 & 0 & B\end{array}\right],\quad \eta_s=\left\{\begin{array}{c}u_g\\ \hline w\\ \hline y_g\end{array}\right\}$$

The goal of optimal control is to determine the design variable in controller matrices A, B and C, thus taking the minimum value of the following performance index. /569

$$J_s=\frac{1}{2}E\{y_f^TQ_1y_f+u^TQ_2u\}=\frac{1}{2}t_r(Q_sX_s) \qquad (6)$$

In the equation, E[·] is the mathematically expected operator; $t_r[.]$ is the matrix track operator; Q_1 is the semi-positive definite weighted matrix, and Q_2 is the positive weighted matrix.

$$Q_a=\begin{bmatrix} H_1^T Q_1 H_1 & 0 \\ 0 & C^T Q_2 C \end{bmatrix}, \quad X_a = E(x_a x_a^T)$$

For iterative substitutional convergence and the closed-loop stability of the control system when solving for the control law, and to allow the control system to have a better asymptotic stability property, the paper applies the concept of the pole arrangement to revise the inequality limiting equation introduced by the above-mentioned optimal output control.

$$g_s(A, B, C)=\max_i(\lambda_i(F_a))<-\sigma, \quad \sigma>0 \qquad (7)$$

In the equation, $\lambda_i(F_a)$ is the i-th eigenvalue of F_a; $g_s(A, B, C)$ indicates the maximum real-number part of the closed-loop system eigenvalue. Therefore, inequality (7) corresponds to the necessary minimum stability allowance required for the control system.

As mentioned above as for the optimized problem (Eq. (6)) with inequality constraint Eq. (7), the feasible direction method can be used for solving it. First, the Lagrange method is used to process the equality constraint, obtaining the necessary condition for minimum J_s as follows:

$$\left.\begin{aligned} F_a X_a + X_a F_a^T + G_a R_a G_a^T = 0 \\ F_a^T \Lambda + \Lambda F_a + Q_a = 0 \end{aligned}\right\} \qquad (8)$$

In the equations, A is the semi-positive definite Lagrange operator matrix,

$$R_a=\begin{bmatrix} R_{a_1} & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & R_{a_2} \end{bmatrix}$$

Obtain matrix X_c; the partitioned matrix is designated as

$$X_s=\left[\begin{array}{c|c} X_f & X_{fs} \\ \hline X_{fs}^T & X_s \end{array}\right],\quad \Lambda=\left[\begin{array}{c|c} \Lambda_f & \Lambda_{fs} \\ \hline \Lambda_{fs}^T & \Lambda_s \end{array}\right]$$

Then, various matrix derivatives of the target function J_s can be obtained

$$\left.\begin{aligned} \frac{\partial J_s}{\partial A} &= \Lambda_{fs}^T X_{fs}+\Lambda_s X_s \\ \frac{\partial J_s}{\partial B} &= (\Lambda_{fs}^T X_f+\Lambda_s X_{fs}^T)H_f^T+\Lambda_s B R_f \\ \frac{\partial J_s}{\partial C} &= Q_2 C X_s+G_s^T(\Lambda_f X_{fs}+\Lambda_{fs} X_s) \end{aligned}\right\} \qquad (9)$$

IV. Design Method of Active Flutter Suppressed Control Law

1. Design variable. In equation (4) for the dynamic output feedback controller, there are only $N_c(m+r)$ independent parameters [1] for the free design of variables. For a smaller number of orders and better dynamic characteristics, the authors consider all feedback circuits with dynamic sectors of the same filter and phase shifter; that is, the controller equation (4) is revised to become:

$$\left.\begin{aligned} \dot{x}_s &= Ax_s+(B_0 B_c)y_f \\ u_s &= Cx_s \end{aligned}\right\} \qquad (10)$$

In the equations, B_o is the vector of N_c-dimension row; B_c is the vector of r-dimension column. /570

To reduce difficulties in realizing the control law, the authors combined the frequency band method from the classical control law, and the above-mentioned design method of the optimal output feedback control law. That is, first they relied on the filter actually required and used in the control law to predetermine all elements of matrices A, B_o and C. Then, matrix B_c is taken as the free designed parameter set to be determined by applying the suboptimal method as proposed herein. Then the matrix derivative equation (9) of the target function is simplified into the matrix derivative.

$$\frac{dJ_s}{dB_s} = B_0^T(\Lambda_{s+1}^T X_s + \Lambda_s X_{s+1}^T)H_s^T + (B_0^T\Lambda_s B_0)B_0 R_{y_0}^{-1} \qquad (11)$$

2. Method of calculating suboptimal output control law: The calculation flow chart [3] of the suboptimal output feedback control law equation (11) is as follows:

(a) Preselect the initial parameter $B_{e,\,0}$, so that $F_{e,o}$ tends to be asymptotically stabilized.
Designate $\sigma_0 = -g_s(B_{s,0})$, and let i = 0.

(b) The iterative substitutional form of the Lyapunov set of equations (8) is:

$$\left.\begin{aligned} F_{s,i}X_{s,i+1} + X_{s,i+1}F_{s,i}^T + G_{s,i}R_s G_{s,i}^T = 0 \\ F_{s,i}^T\Lambda_{i+1} + \Lambda_{i+1}F_{s,i} + Q_{s,i} = 0 \end{aligned}\right\} \qquad (12)$$

Determine $X_{e,i+1}$ and A_{i+1}.

(c) From equation (11), let $(dJ_s/dB_a) = 0$ and solve for the new control law

$$B_{s,new} = -(B_0^T\Lambda_{s,i+1}B_0)^{-1}B_0^T(\Lambda_{s,i+1}^T X_{s,i+1} + \Lambda_{s,i+1}X_{s,i+1}^T)H_s^T R_{y_0}^{-1} \qquad (13)$$

and define the matrix gradient

$$d(B_{s,i}) = B_{s,new} - B_{s,i} \qquad (14)$$

(d) Calculate

$$B_{s,i+1} = B_{s,i} + \alpha_{i+1}d(B_{s,i}) \qquad (15)$$

In the equation, the value of the constant α_{i+1} is taken in the sector (0, 1), let $B_{e,I+1}$ satisfy the iterative substitutional form of Equation (7) as:

$$g_s(B_{s,i+1}) < -\sigma_i \qquad (16)$$

(e) Calculate the performance index

$$J_{s,i+1} = \frac{1}{2}t_r(Q_s X_{s,i+1})$$

and norm

$$\delta_{i+1} = \left\| \frac{dJ_s}{dB_{s,i}} \right\|$$

If

$$\left|\frac{J_{s,i+1}-J_{s,i}}{J_{s,i}}\right|<\varepsilon_1,\quad \left|\frac{\delta_{i+1}-\delta_i}{\delta_i}\right|<\varepsilon_2$$

then the calculation ceases, and one adopts the suboptimal output feedback control law $B_{s,sub}=B_{s,i+1}$

Otherwise, let $\sigma_{i+1}=-g_s(B_{s,i+1})$, $i=i+1$, and continue the calculation after step (b).

3. Slowly-varying parametric design method [3] of active flutter suppressed control law

At present, in research on active flutter suppression, the method of frozen coefficients is adopted in all cases by fixing a certain flight velocity V_d (generally called the design point, usually higher than the open-loop flutter velocity) in designing the control law. This method leads to difficulty in preselecting initial value of control law in order to gradually stabilize the control system. To solve this problem, this paper proposes the following design method of slowly-varying parameters according to characteristics of an aerodynamic elastic system:

First step, select the highest design velocity V_d (greater than the open-loop flutter velocity V_{t_o}); /571

Second step, let $V_1< V_{t_o}$; at that time the system open loop is stabilized. Therefore, the initial value of control law is $B_{c,o}^{(1)}=0$; then by applying the design method of the suboptimal output control law presented by the paper, one solves for the suboptimal control law $B_{c,sub}^{(1)}$, and designated j=1;

Third step, let $V_{j+1}\in(V_j, V_d)$, so that at velocity V_{j+1}, under control law the aerodynamic system gradually stabilizes as acted on by $u(s)=C(sI-A)^{-1}B_o\cdot B_{c,sub}^{(j)}$. Therefore the corresponding initial value of the control law can be taken as $B_{c,o}^{(j+1)}=B_{c,sub}^{(j)}$, and one obtains the new control law $B_{c,sub}^{(j+1)}$ at velocity V_{j+1}.

Fourth step, if $V_{J+1}=V_d$, the calculation ceases, and one adopts the suboptimal output feedback control law of the closed-loop control system in the entire flight envelope as

$$B_{c,sub}=B_c^{(j+1)}$$

Otherwise, designate

$$B_{c,sub}^{(j)}=B_c^{(j+1)}$$

and let $V_j=V_{j+1}$, $j=j+1$; then continue in step (c).

V. Synthesis and Analysis of Active Suppression Control Law of Wind Model Flutter

Figure 4 shows the open-loop acceleration frequency response curves measured at point 1 of the wing (refer to Fig. 1) during wind tunnel flutter test when the air velocity is 20 meters per second. To reduce a frequency response of various orders of mode status following the three-order mode-status frequency, the authors selected a four-order low pass filter

$$C(sI-A)^{-1}B_0=\frac{40^4\times 0.18(s+100)^2}{(s^2+40s+40^2)(s+30)(s+60)} \qquad (17)$$

Then, record the acceleration signals at three measurement points 1, 2 and 3 as the output vector y_s of the servoaerodynamic elastic system, and take design parameters of the output and input weighted matrices as

$$\left.\begin{aligned} Q_1&=diag(0.01,\ 0.01,\ 0.01)\\ Q_2&=0.01\\ R_{w_g}&=0.0001\\ R_w&=0.1\\ R_{y_g}&=diag(0.001,\ 0.001,\ 0.001)\end{aligned}\right\} \qquad (18)$$

Apply the slowly-varying parametric design method of the above-mentioned suboptimal output feedback control law, and derive the suboptimal control law as follows (V_d=25 m/s):

$$B_{c,sub}=(-1.3878,\ 3.4235,\ 0.8380) \qquad (19)$$

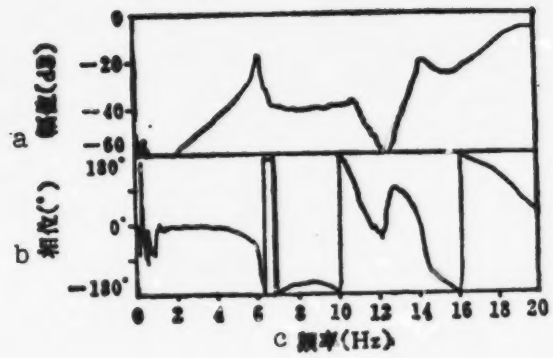

Fig. 4. Acceleration Response Curve (V=20 m/s) at Point 1 of the Wing. Key: a. Amplitude value; b. Phase position; c. Frequency.

Refer to Fig. 5 for the closed-loop locus of the wing model with the above-mentioned control law shown as Eqs. (17) and (19). As found by calculation, the closed-loop flutter velocity V_1=29 m/s. Compared to the open-loop situation, the flutter velocity is about 45 percent higher. In addition, the coupled oscillation of aileron elastic deflection and one-order wing bending during an open loop originally have been eliminated. The closed-loop flutter state is one-order wing turning. /572

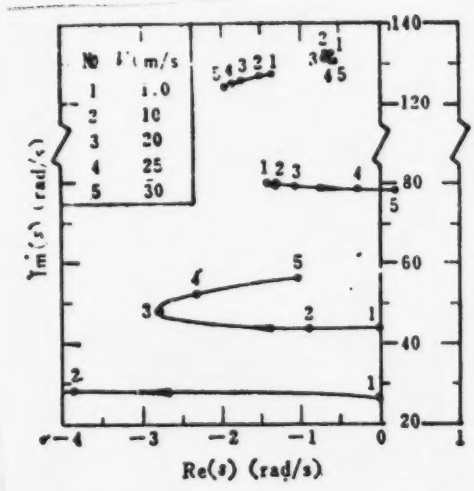

Fig. 5. Closed-loop velocity root-locus diagram.

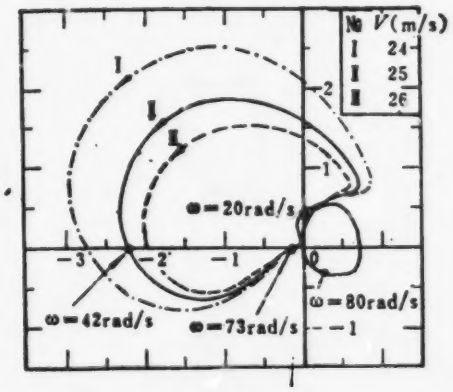

Fig. 6. Nyquist curves of control system (arrows represent the direction of higher frequencies).

Figure 6 shows Nyquist curves of the closed-loop control system for three different air flow velocities. At the design velocity V_d=25 m/s (about 21 percent higher than the open-loop flutter velocity), there are increment allowances of -17.72 dB and +6.85 dB, and phase allowances of -130° and +30.1°. With decreasing velocity, the steady allowance of the closed-loop control system is slowly enhanced. As seen in Fig. 6, the control system has better steady allowance within the entire flight envelope, thereby meeting the design requirements imposed on the flutter active suppressed control law.

VI. Conclusion

The paper combines the frequency band method from classical control theory and the optimal control theory in modern control theory by proposing a slowly varying parametric design method by determining the suboptimal output feedback control law. In applying this method, one designs the flutter active suppression control law, thereby solving the initial-value problem of the control law, the problem of iterative substitutional convergence, ensuring the closed-loop stability of the control system. Under this method, few sensors are used to measure signals as feedback; the derived control law is readily feasible. In this paper, a delta wing model is used as the research object; by applying this method, the flutter active suppression control law is designed. As indicated by calculation results, the wing flutter rate can be increased by 45 percent; in addition, the control system has a better stability allowance in the entire flight envelope.

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The paper was received for publication on 18 January 1986.

APPENDIX A

Status and output equation of servoaerodynamic elastic system:

Motion equation (under disturbance) of wing model is

$$(M_s s^2+C_s s+K_s)x_s(s)+M_{s\beta}\beta(s)$$
$$-q\left((A_0:D_0:E_0)+(A_1:D_1:E_1)\bar{s}+(A_2:D_2:E_2)\bar{s}^2+\sum_{i=3}^{6}\frac{\bar{s}}{\bar{s}+r_i}(A_i:D_i:E_i)\right)\begin{Bmatrix} x_s(s) \\ \beta(s) \\ \dfrac{w_g(s)}{V} \end{Bmatrix} \qquad \text{(A-1)}$$

From Eq. (1), the rudder mechanism transfer function, the status equation of the rudder mechanism

$$\dot{x}_{act}=A_{act}x_{act}+B_{act}u \qquad \text{(A-2)}$$

is obtained. In the equation,

$$x_{act}=\begin{Bmatrix} \beta \\ \dot{\beta} \\ \ddot{\beta} \end{Bmatrix}$$

However, the air flow disturbance is expressed by the Dryden model as a Markov process:

$$\left.\begin{aligned} \dot{x}_g&=A_g x_g+B_g w \\ \begin{Bmatrix} w_g/V \\ \dot{w}_g/V \\ \ddot{w}_g/V \end{Bmatrix}&=C_g x_g \end{aligned}\right\} \qquad \text{(A-3)}$$

By introducing the vector of the augmented aerodynamic state, thus

$$x_{wi}=\frac{q\bar{s}}{\bar{s}+r_i}(A_i x_s(s)+D_i\beta(s)+E_i w_g(s)/V)$$
$$i=3,\ 4,\ 5,\ 6 \qquad \text{(A-4)}$$

Merging the above-mentioned equations, one obtains a status equation

$$\dot{x}_s = F_s x_s + G_u u + G_w w \qquad \text{(A-5)}$$

In the equation,

$$x_s = (x_p^T,\ \dot{x}_p^T,\ x_{v1}^T, \cdots, x_{vn}^T,\ x_{\omega}^T,\ x_g^T)^T$$

$$F_s = \begin{bmatrix} F & B_v & B_\omega C_g \\ 0 & A_{vv} & 0 \\ 0 & 0 & A_g \end{bmatrix},\quad G_u = \begin{bmatrix} 0 \\ B_{vv} \\ 0 \end{bmatrix},\quad G_w = \begin{bmatrix} 0 \\ 0 \\ B_g \end{bmatrix}$$

$$M = M_p - \frac{1}{2}\rho b_0^3 A_3$$

$$C' = C_p - \frac{1}{2}\rho V b_0 A_1$$

$$K' = K_p - \frac{1}{2}\rho V^2 A_0$$

$$F = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ -M^{-1}K' & -M^{-1}C' & M^{-1} & \cdots & M^{-1} \\ 0 & qA_1 & -r_1\frac{V}{b_0} I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & qA_n & 0 & \cdots & -r_n V/b_0 I \end{bmatrix}$$

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$$B_v = \begin{bmatrix} 0 & 0 & 0 \\ qM^{-1}D_0 & \frac{1}{2}\rho V b_0 M^{-1}D_1 & \frac{1}{2}\rho b_0^2 M^{-1}D_2 - M^{-1}M_{p\theta} \\ 0 & qD_1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & qD_n & 0 \end{bmatrix}$$

$$B_\omega = \begin{bmatrix} 0 & 0 & 0 \\ qM^{-1}E_0 & \frac{\rho V b_0}{2} M^{-1}E_1 & \frac{\rho b_0^2}{2} M^{-1}E_2 \\ 0 & qE_1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & qE_n & 0 \end{bmatrix}$$

Assume that r acceleration sensors are used to measure the elastic motion signals of the wing, and considering the measurement of noise y_o, the output equation of the wing is obtained

$$y_1 = H_1 x_1 + y_0 \qquad \text{(A-6)}$$

In the equation,

$$H_1 = C_1 M^{-1}(-K' \vdots -C' \vdots I \vdots I \vdots I \vdots I \vdots D_0 \vdots D_2 C_2)$$

C_p is the oscillation-type parametric matrix of the measurement point.

$$D_0 = \left[\frac{1}{2}\rho V^2 D_0 \;\middle|\; \frac{1}{2}\rho V b_0 D_1 \;\middle|\; \frac{1}{2}\rho b_0^2 D_2 - M_{p0}\right]$$

$$D_2 = \left[\frac{1}{2}\rho V^2 E_0 \;\middle|\; \frac{1}{2}\rho V b_0 E_1 \;\middle|\; \frac{1}{2}\rho b_0^2 E_2\right]$$